Title: ParameTRICKS

Brief Overview:

This lesson is primarily a graphing activity in which students will manipulate equations of a circle in order to translate the center to another point. They will also manipulate the T-step to obtain different polygons, and by changing the value of T in the equation they will rotate the polygon.

Link to Standards:

- **Problem Solving** Students will predict changes in the equation from changes in the graph. They will match graphs and data. Students will use their tables to solve problems.

- **Algebra** Students will develop parametric equations for their data. Students will have the opportunity to make the connections between graphing in the rectangular coordinate system and parametrics.

- **Reasoning** Students will look for patterns by interpreting graphs, make conjectures about the central angles of polygons, and write a set of parametric equations for the polygon.

Grade/Level:

Grades 9-12

Duration/Length:

This lesson will take 1-2 periods (90 min.).

Prerequisite Knowledge:

Students should be familiar with the concept of parametric equations, and using the TI-82 calculator. They should have experience with entering data, graphing, adjusting windows, adjusting mode and format, and identifying polygons.

Objectives:

Students will:

- write the equation of a circle in parametric form.
- manipulate the equation of a circle with center (0,0) so that the center is new point (h,k).
- use the equation of a circle and the appropriate windows to graph and obtain regular polygons.
- match equations of circles to given graphs.
- rotate points on a graph.

Materials/Resources/Printed Materials:

- TI-83 graphics calculator
- Student Worksheets (3 activities)
- Assessment sheet

Development/Procedures:

- Student will read the activity sheet.

- Teacher will guide students through the first problem of each activity.

- Students will solve problems working in pairs, while the teacher circulates and monitors their progress.

- Teacher will only offer additional assistance to groups that fail to arrive at an appropriate conclusion.

Evaluation:

At a group level, students will sketch graphs from given data and complete questions and tables on worksheets. They will be given a quiz to determine their understanding individually.

Extension/Follow Up:

1. Students could obtain shapes other than regular polygons by manipulating the window where the T-step is not a factor of 360.
2. As the number of sides increase, a polygon will approach a circle. By finding areas of these polygons, students can discover pi. {radius is a constant of 1}
3. Rotation of shapes through given angles.
4. Discussions of position, direction, and speed of a particle at time t.
5. Reflection of shape about the axes.

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Activity 1

Going in Circles

The two basic circular functions, *sine* and *cosine*, are defined using a circle of radius r centered at the origin. In the figure, a length t is measured counter-clockwise around the circle from the point (r,0) to the point P, giving the angle of t radians at the origin. If P has coordinates (x,y), then we define

$x = r \cos t$ x and y are parametric
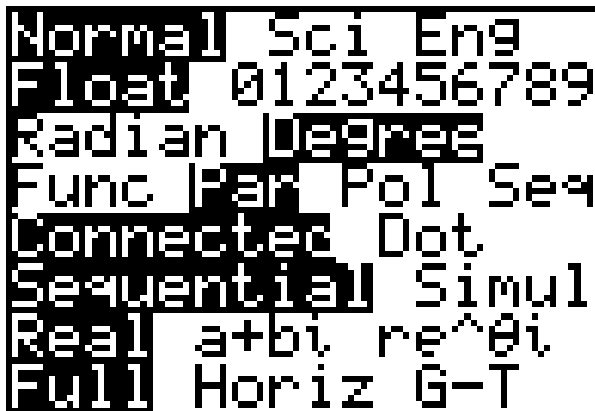$y = r \sin t$ equations of the circle
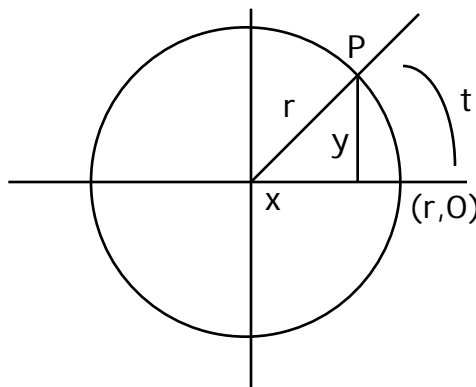
1. Give the parametric equations for a circle with radius 6 and center (0,0).

$X_{1T} = $ _____

$Y_{1T} = $ _____

In order to graph these on your calculator you must have your calculator in the parametric mode.

Press $\boxed{y =}$ and enter both equations.

```
Plot1  Plot2  Plot3
\X1T■6cos(T)
 Y1T■6sin(T)
\X2T=
 Y2T=
\X3T=
 Y3T=
\X4T=■
```

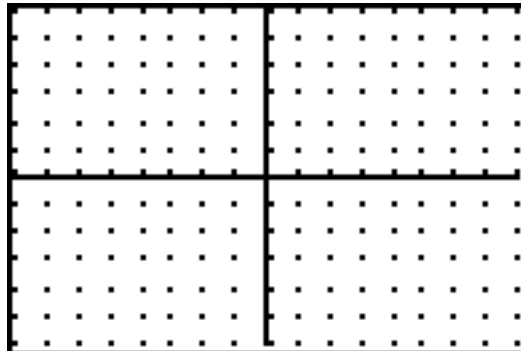Using the following window

```
WINDOW
 Tmin=0
 Tmax=360
 Tstep=5
 Xmin=-10
 Xmax=10
 Xscl=1
↓Ymin=-10
```
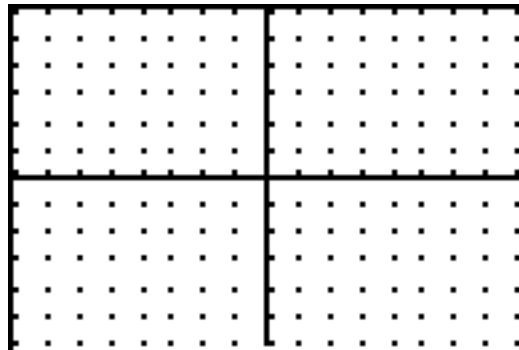$Y_{max} = 10$

Press **ZOOM** 5.

Sketch your graph:

2. Give the parametric equations and sketch the graph for a circle with center at the origin and radius 2 3 units.
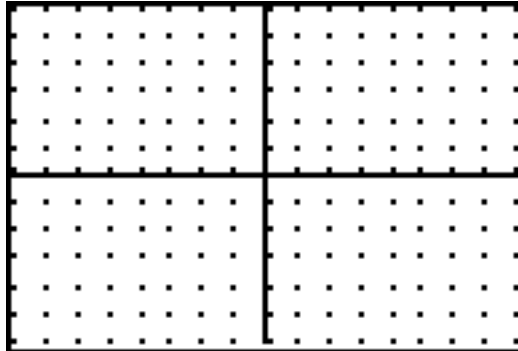
$X_{1T} = $ _____

$Y_{1T} = $ _____

3. Graph the following circle: $X_{1T} = 6\cos t + 2$
$Y_{1T} = 6\sin t + 4$

Sketch the graph:

What is the center of the new circle? _____

CLEAR ALL EQUATIONS!

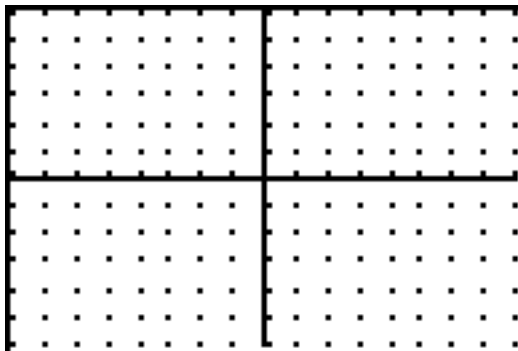4. Graph the following circles.

Circle 1   Circle 2

$X_{1T} = 3\cos t$ $X_{2T} = 3\cos t + 3$

$Y_{1T} = 3\sin t$ $Y_{2T} = 3\sin t + 4$

State the radii and centers of the circles.

Circle 1: radius _____   Circle 2: radius _____

center _____   center _____

5. The equations for the circle with center (0,0) and radius 6 are:

$$X_{1T} = 6\cos t \qquad Y_{1T} = 6\sin t.$$

Write the equations for the circle with the same radius but new center at (5,4).

$$X_{2T} = \underline{\hspace{3cm}}$$

$$Y_{2T} = \underline{\hspace{3cm}}$$

6. Complete the table:

radius	center	$X_{1T} = ?$	$Y_{1T} = ?$
2	(0,0)		
7	(-2,4)		
3	(-8,9)		
5	(8,-2)		
6	(0,-9)		
r	(a,b)		

Activity 2

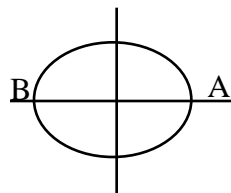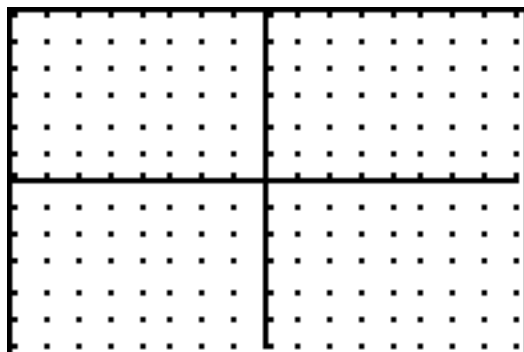**Placing Equidistant Points on a Circle**

<u>Review</u>: Your calculator must be set for the following modes:

```
Normal Sci Eng
Float 0123456789
Radian Degrees
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```

1. Graph the circle of radius 6 and center (0,0) with T-step = 5.
    Be sure to press **ZOOM** 6, then **Zoom** 5.

X_{1T} = _____ Y_{1T} = _____

Press **TRACE**, the coordinates of A are (___, ____), T = _____
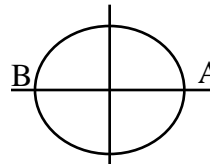
Press ⟶ and move pixel to B, coordinates of B are (___, ____), and T = _____

INSERTING TWO POINTS

Press 2nd **FORMAT**, arrow down to **AXES OFF** and press **ENTER**. Press **GRAPH**

SETTING T-STEP:

 Between points A and B, T has changed by 180˚.
 Press **MODE** and arrow down to **DOT**.

WHAT IS T STEP? This is the change in T between two consecutive points.
 The T-step for AB is 180˚.

Press **WINDOW** , change T-step to 180˚.
Press **GRAPH**.

What do you see?

Verify the points. Press ⎡TRACE⎤ the coordinates of A (_____,_____)
 Press ⎡ ⟶ ⎤ the coordinates of B (_____, _____)

INSERTING 3 POINTS

Change T-Step to 120.
Press **GRAPH** and note your observation. What do you see?

Press **MODE** and arrow down to **CONNECTED**.   Press **GRAPH**. What do you see?

Working with your partner, compete the table below.

T-steps	Number of points	Name of polygon
120		
90		
72		
60		
$\frac{360}{7}$		
45		
40		
36		

Let Y = T-step
Let X = number of dots.
Write an equation for Y in terms of X.   Y = _____

SET UP A T-STEP TABLE

Press **MODE**, arrow down to **FUNC** and press **ENTER**.
Press $\boxed{Y=}$.   Clear all equations.
Enter Y_1 = 360 / x.
Press 2nd **TBLSET** and set up table as shown.

```
TABLE SETUP
 TblStart=1■
 ΔTbl=1
Indpnt: Auto  Ask
Depend: Auto  Ask
```

Press 2nd **TABLE**

Use your table to complete.

Number of sides	T-Step
13	_____
_____	16.364
28	_____
_____	14.4
_____	10.286

What is the T-Step for a polygon with 59 sides?_____

CHALLENGE! What shape do you observe as the number of points increase?_____

Activity 3

POLYEXERCISE - ROTATING POLYGONS

I. Inscribing an equilateral triangle in a circle with radius 6 units..

Review: The parametric equations for the circle are:

$$X_{1T} = \text{_____} \qquad Y_{1T} = \text{_____}$$

$$\text{T- step} = \text{_____}$$

   a. Turn on the calculator.
   b. Turn axes on by pressing 2nd **FORMAT** arrow down to **AXES**, press **ENTER**.
   c. You will need to be in the parametric mode. Press **MODE**, arrow down to **PAR**, and
        press **ENTER**. Be sure at this time that **CONNECTED** and **DEGREE** are also
        highlighted.
   d. Press $\boxed{Y=}$ and clear all equations.
   e. Enter the equations: $X_{1T} = 6 \cos T \qquad Y_{1T} = 6 \sin T$
   f.   Press **ZOOM** 6.
   g. Adjust T-step to 120 (T-step is in **WINDOW**)
   h. Press **ZOOM 5**.

II. Labelling the Triangle:

   Press TRACE   pixel is at T = 0,   name this point A.

   Press $\boxed{\longrightarrow}$ pixel is at T = 120,   name this point B

   Press $\boxed{\longrightarrow}$ pixel is at T= 240,   name this point C
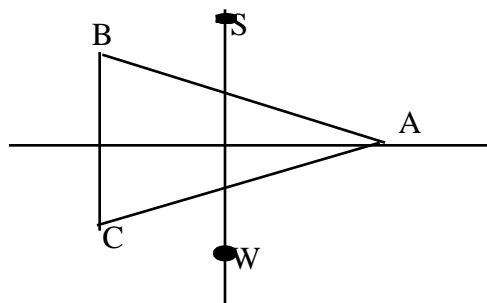
III. Moving B to the positive Y axis   (That is to the line containing the point S)

In moving from B to S we are moving from T=120 to T = 90. We are going back 30°, that is T had been reduced by 30°. Our new value for T is (T - 30).
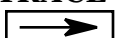
Our new parametric equations are:

$$X_{1T} = 6 \cos (T - 30) \qquad Y_{1T} = 6 \sin (T - 30)$$

 DOES IT WORK?

Enter your equations and graph.

Is B in the correct position?

Press **TRACE**
Press $\longrightarrow$ (T = 120   is on the positive Y-axis)

IV. Moving B to the negative X-axis   (to R)

At B   T= 120
At R   T = 180

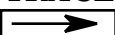In moving from B to R, T increases by 60 degrees , therefore we will need to add 60 degrees to T.

Our new parametric equations are: $X_{1T} = 6 \cos (T + 60) \qquad Y_{1T} = 6 \sin (T + 60)$

Press $\boxed{Y =}$
Enter your parametric equations and graph.

Is B in the correct position?

Press **TRACE**
Press $\longrightarrow$ (T120 is on the negative x axis)

V. Moving vertex C to the negative Y axis (in the same line with the point W).

The value of T at C = _____

The value of T at W = _____

Therefore the change in T is _____

Our new parametric equations are $X_{1T} =$ _____ $Y_{1T} =$ _____

If C is in the correct position, then T = ____ must be on the negative y - axis.

VI. a. Graph: $X_{1T} = 8 \cos T$ $Y_{1T} = 8 \sin T$

Using standard window with T-step = 72.
(Remember to Press **ZOOM** 6, **ZOOM** 5.)

b. Sketch your graph.

c. Press **TRACE** and label the vertices as follows.

(Press [→] to continue)   at T = 0,   A = _____

at   T = 72.   B = _____

at   T = 144   C = _____

at   T = 216   D = _____

at   T = 288   E = _____

d. Move vertex B to the positive Y axis.

Value of T at B = _____

Value of T on the positive y axis = _____

Change in T = _____

The new parametric equations are: X_{1T} =_____

Y_{1T} =_____

Coordinates of A= _____   B = _____   C = _____

D = _____   E = _____

CHALLENGE: Can you find the equations necessary to obtain the following graph?
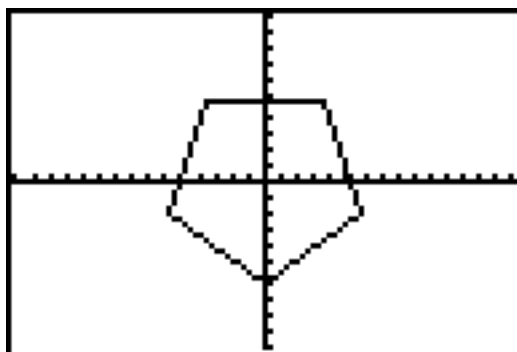
ASSESSMENT

Quiz   Name _____
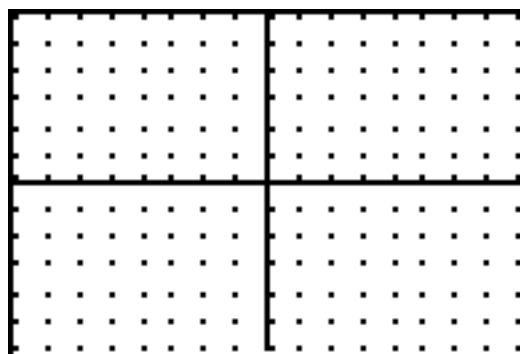   Period _____
                                        Date_____

Part A:

1. Write the parametric equations of the circle with center (0,0) and radius 3 5. Sketch the
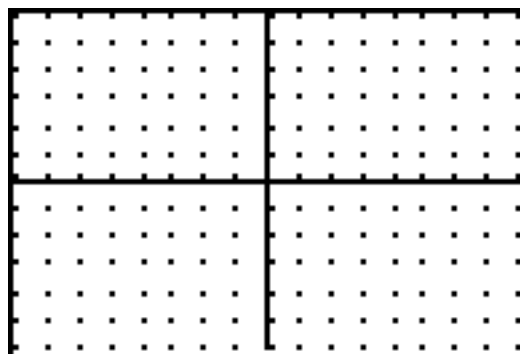   graph.

$$X_{1T} = \underline{\hspace{3cm}}$$

$$Y_{1T} = \underline{\hspace{3cm}}$$

2. Give the parametric equation of a circle with center (3,4) and radius 5 units. Sketch the
    graph.

$$X_{1T} = \underline{\hspace{3cm}}$$

$$Y_{1T} = \underline{\hspace{3cm}}$$

3. The parametric equations of a circle of radius r and center (0,0) are:

$$X_{1T} = \underline{\hspace{3cm}}$$
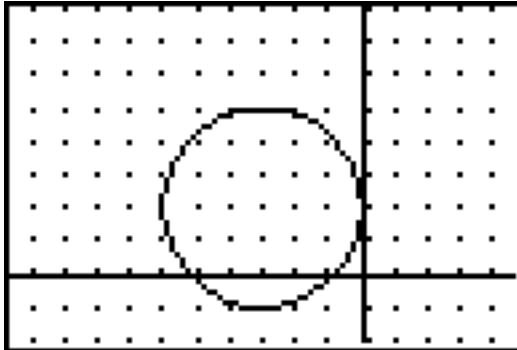$$Y_{1T} = \underline{\hspace{3cm}}$$

If we need to translate the center to the point (h,k). Then our parametric equations become:

$$X_{1T} = \underline{\hspace{2.5cm}} \qquad Y_{1T} = \underline{\hspace{3cm}}$$

Part B:

Describe each circle and write the corresponding parametric equations.
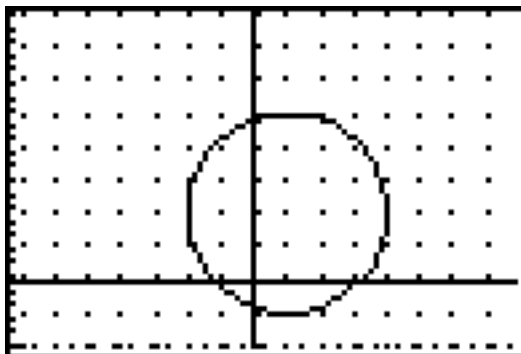
4.

radius =_____
center = _____

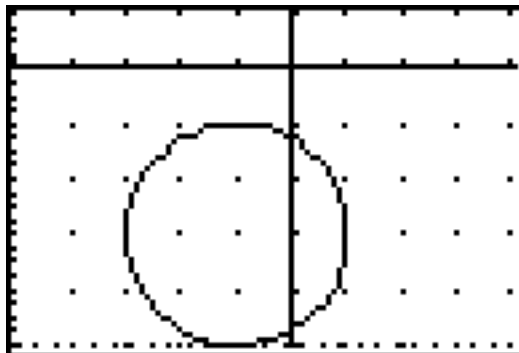$X_{1T} =$ _____ $Y_{1T} =$ _____

5.

radius = _____
center = _____

$X_{1T} =$ _____ $Y_{1T} =$ _____

6.

radius = _____
center = _____

$X_{1T} =$ _____ $Y_{1T} =$ _____

7. Write the equations of a circle of radius 8 and center (0,0).

$$X_{1T} = \underline{\hspace{3cm}} \qquad Y_{1T} = \underline{\hspace{3cm}}$$
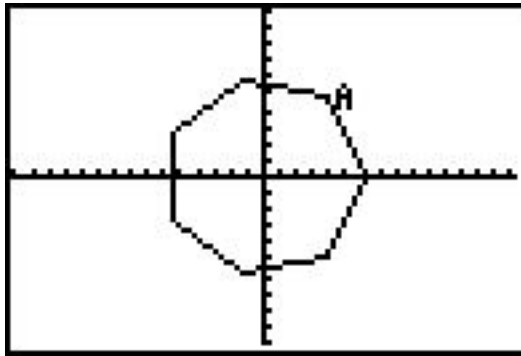
   Inscribe a regular hexagon in this circle.   T-step = _____

8. Inscribe a square in problem 1. T-step = _____.

9. A T-step of 45 will inscribe a regular hexagon in a given circle.   True/ False.

10. A T-step of _____ will inscribe a 14 sided regular polygon in a given circle.

11. Write the parametric equations for the graph shown below.

$$X_{1T} = \underline{\hspace{3cm}} \qquad Y_{1T} = \underline{\hspace{3cm}}$$

T-step =_____

Move vertex A to the positive y axis. The new parametric equations are:

$$X_{1T} = \underline{\hspace{3cm}} \qquad Y_{1T} = \underline{\hspace{3cm}}$$

NOTES TO TEACHER

<u>ACTIVITY I</u>:

    A. Center of Circle: x coordinate of center is at T = 90

                                           y coordinate of center is at T = 0.

    2. Pixel will only move between 0 and 360. If your arrow passes a given point, you will have to arrow back to that point.

<u>ACTIVITY II</u>:

To obtain points you have to be in **dot** mode. To obtain the polygons you have to be in **connected** mode.

<u>ACTIVITY III</u>:

To highlight T on the TI - 82 you have to press **TRACE** ⬆️ .

In all activities where T is needed on the TI - 82, replace Press **TRACE** with

press **TRACE** ⬆️ .

To turn on the axes on the TI-82, press WINDOW and arrow over to FORMAT.

For graphs 3 (Activity I) and VI (Activity III): in sketching graphs, it will be necessary for each tic mark to represent 2 units.